$\left.\begin{array}{|l|l|l|l|l|l|l|}\hline \mathbf{1} & \text { (i) } & & \begin{array}{l}\arcsin x=\pi / 6 \Rightarrow x=\sin \pi / 6 \\ =1 / 2\end{array} & \begin{array}{c}\text { M1 } \\ \text { A1 } \\ {[2]}\end{array} & \text { allow unsupported answers }\end{array}\right]$

| 2 (i) $a=0, b=3, c=2$ | $\mathrm{~B}(2,1,0)$ | or $a=0, b=-3, c=-2$ |
| :--- | :--- | :--- |
| (ii) $a=1, b=-1, c=1$ <br> or $a=1, b=1, c=-1$ | $\mathrm{~B}(2,1,0)$ |  |



| 4(i) period $180^{\circ}$ | B1 <br> [1] | condone $0 \leq x \leq 180^{\circ}$ or $\pi$ |
| :--- | :--- | :--- |


| $\begin{array}{ll} \hline 5 \text { (i) } & \text { bounds }-\pi+1, \pi+1 \\ \Rightarrow & -\pi+1<\mathrm{f}(x)<\pi+1 \end{array}$ | B1B1 B1cao <br> [3] | or $\ldots<y<\ldots$ or ( $-\pi+1, \pi+1$ ) | not $\ldots$. $<x<\ldots$, not 'between ...' |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { (ii) } y=2 \arctan x+1 x \leftrightarrow y \\ & \Rightarrow \quad \frac{x=2 \arctan y+1}{2}=\arctan y \\ & \Rightarrow \quad y=\tan \left(\frac{x-1}{2}\right) \Rightarrow \mathrm{f}^{-1}(x)=\tan \left(\frac{x-1}{2}\right) \end{aligned}$ | M1 <br> A1 <br> A1 <br> B1 <br> B1 <br> [5] | attempt to invert formula or $\frac{y-1}{2}=\arctan x$ <br> reasonable reflection in $y=x$ $(1,0)$ intercept indicated. | one step is enough, i.e. $y-1=2 \arctan x$ or $x-1=2 \arctan y$ <br> need not have interchanged $x$ and $y$ at this stage <br> allow $y=\ldots$ <br> curves must cross on $y=x$ line if present (or close enough to imply intention) curves shouldn't touch or cross in the third quadrant |


| $\begin{array}{ll} \mathbf{6 ( i )} & \text { At P, } x \cos 3 x=0 \\ \Rightarrow & \cos 3 x=0 \\ \Rightarrow & 3 x=\pi / 2,3 \pi / 2 \\ \Rightarrow & x=\pi / 6, \pi / 2 \\ & \text { So P is }(\pi / 6,0) \text { and Q is }(\pi / 2,0) \end{array}$ | M1 <br> M1 <br> A1 A1 <br> [4] | or verification $3 x=\pi / 2,(3 \pi / 2 \ldots)$ <br> dep both Ms condone degrees here |
| :---: | :---: | :---: |
| $\begin{aligned} & \text { (ii) } \frac{d y}{d x}=-3 x \sin 3 x+\cos 3 x \\ & \quad \text { At P, } \frac{d y}{d x}=-\frac{\pi}{2} \sin \frac{\pi}{2}+\cos \frac{\pi}{2}=-\frac{\pi}{2} \\ & \text { At TPs } \frac{d y}{d x}=-3 x \sin 3 x+\cos 3 x=0 \\ & \Rightarrow \quad \cos 3 x=3 x \sin 3 x \\ & \Rightarrow \quad 1=3 x \sin 3 x / \cos 3 x=3 x \tan 3 x \\ & \Rightarrow \quad x \tan 3 x=1 / 3^{*} \end{aligned}$ | M1 <br> B1 <br> A1 <br> M1 <br> A1cao <br> M1 <br> E1 <br> [7] | Product rule <br> $\mathrm{d} / \mathrm{d} x(\cos 3 x)=-3 \sin 3 x$ <br> cao (so for $\mathrm{d} y / \mathrm{d} x=-3 x \sin 3 x$ allow B1) <br> mark final answer <br> substituting their $-\pi / 6$ (must be rads) <br> $-\pi / 2$ <br> $\mathrm{d} y / \mathrm{d} x=0$ and $\sin 3 x / \cos 3 x=\tan 3 x$ used <br> www |
| $\text { (iii) } \begin{aligned} & A=\int_{0}^{\pi / 6} x \cos 3 x d x \\ & \text { Parts with } u=x, \mathrm{~d} v / \mathrm{d} x=\cos 3 x \\ & \mathrm{~d} u / \mathrm{d} x=1, v=1 / 3 \sin 3 x \\ & \Rightarrow \quad A=\left[\frac{1}{3} x \sin 3 x\right]_{0}^{\frac{\pi}{6}} \int_{0}^{\pi / 6} \frac{1}{3} \sin 3 x d x \\ &=\left[\frac{1}{3} x \sin 3 x+\frac{1}{9} \cos 3 x\right]_{0}^{\frac{\pi}{6}} \\ &=\frac{\pi}{18}-\frac{1}{9} \end{aligned}$ | B1 <br> M1 <br> A1 <br> A1 <br> M1dep <br> A1 cao <br> [6] | Correct integral and limits (soi) - ft their P , but must be in radians <br> can be without limits <br> dep previous A1. <br> substituting correct limits, dep $1^{\text {st }}$ M1: ft their P provided in radians o.e. but must be exact |



| $\begin{array}{ll} \mathbf{8} \text { (i) } & -\pi / 2<\arctan x<\pi / 2 \\ \Rightarrow & -\pi / 4<\mathrm{f}(x)<\pi / 4 \\ \Rightarrow & \text { range is }-\pi / 4 \text { to } \pi / 4 \end{array}$ | M1 <br> A1cao <br> [2] | $\begin{aligned} & \pi / 4 \text { or }-\pi / 4 \text { or } 45 \text { seen } \\ & \text { not } \leq \end{aligned}$ |
| :---: | :---: | :---: |
| $\begin{array}{ll} \text { (ii) } & y=1 / 2 \arctan x \quad x \leftrightarrow y \\ & x=1 / 2 \arctan y \\ \Rightarrow & 2 x=\arctan y \\ \Rightarrow & \tan 2 x=y \\ \Rightarrow \quad & y=\tan 2 x \\ \text { either } \frac{d y}{d x}=2 \sec ^{2} 2 x \end{array}$ | M1 <br> A1cao <br> M1 <br> A1cao | $\tan (\arctan y$ or $x)=y$ or $x$ <br> derivative of $\tan$ is $\sec ^{2}$ used |
| $\text { or } \begin{aligned} y=\frac{\sin 2 x}{\cos 2 x} \Rightarrow \frac{d y}{d x} & =\frac{2 \cos ^{2} 2 x+2 \sin ^{2} 2 x}{\cos ^{2} 2 x} \\ & =\frac{2}{\cos ^{2} 2 x} \end{aligned}$ | M1 <br> A1cao | quotient rule <br> (need not be simplified but mark final answer) |
| When $x=0, \mathrm{~d} y / \mathrm{d} x=2$ | $\begin{aligned} & \mathrm{B} 1 \\ & {[5]} \end{aligned}$ | WWW |
| (iii) So gradient of $y=1 / 2 \arctan x$ is $1 / 2$. | $\mathrm{B} 1 \mathrm{ft}$ [1] | ft their ' 2 ', but not 1 or 0 or $\infty$ |


|  |  |  |  |
| :--- | :--- | :--- | :--- |
|  | $x=1 / 2$ <br> $\cos \theta=1 / 2$ | B1 |  |
|  | $\Rightarrow \quad$ | M1 |  |
|  |  | A1 |  |
|  |  | M1A0 for $1.04 \ldots$ or $60^{\circ}$ |  |

